CDF Note 8955

Measurement of the top quark mass in the dilepton channel using the Neutrino Weighting Algorithm at the CDF-II detector with $1.8\ \mathrm{fb}^{-1}$ of data.

The CDF Collaboration
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We report a of the top quark mass measurement in the dilepton channel of $t\bar{t}$ events from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. This measurement uses a dataset with integrated luminosity of $1.8\ \mathrm{fb}^{-1}$, containing 124 $t\bar{t}$ candidates. We employ Neutrino Weighting Algorithm to reconstruct events. Monte Carlo templates of the reconstructed top quark mass are produced as a function of the true top quark mass. The distribution of reconstructed top quark mass from data is compared to the Monte Carlo templates using a likelihood fit to obtain: $\mathrm{M_{top}} = 172.0\ ^{+6.1}_{-6.0}\ \mathrm{GeV}/c^2$

I. INTRODUCTION

The top quark mass is a fundamental parameter of the Standard Model, and plays an important role in the the precise prediction of electroweak observables such as the Higgs boson mass. The radiative corrections of many electroweak observables are dominated by the large top quark mass. Furthermore, a large value of the top quark mass indicates a strong Yukawa coupling to Higgs, and could be a sign for a special role of the top quark in the understanding of electroweak symmetry breaking [2]. Thus, a precise measurement of the top quark mass provides a crucial test of the consistency of the Standard Model and could help constrain physics beyond the Standard Model. Measurements using different decay channels can also provide important clues for possible Standard Model extensions. In this paper, we report a measurement of the top quark mass with the CDF-II detector [1], using the data sample from March 2002 to March 2007 runs, corresponding to a total integrated luminosity of $1.8\ \mathrm{fb}^{-1}$.

At the Tevatron, top quarks are produced primarily as $t\bar{t}$ pairs and decay to W bosons and b quarks nearly 100% of the time within the Standard Model. Then, the W bosons can decay into lepton-neutrino ($l\nu$) or quark pairs ($q\bar{q}'$). In this measurement, we use the "dilepton" channel of $t\bar{t}$ candidates in which both W bosons decay to $l\nu$ pairs, where the lepton is either an electron or muon.

The template method relies on good Monte Carlo (MC) modeling of $t\bar{t}$ and background events. We generate a set of Monte Carlo samples at a range of true top quark masses ($\mathrm{M_{top}}$). Using Neutrino Weighting Algorithm (NWA) [3], [4], we form an estimator of the true top quark mass: reconstructed top mass m_t^{reco}. We compute the m_t^{reco} distributions or "templates" from all the $\mathrm{M_{top}}$ samples. Kernel density estimation (KDE) is used to compute probability density function for m_t^{reco} for each MC signal sample. Local Polynomial Smoothing is used to calculate the probability density function for an arbitrary value of $\mathrm{M_{top}}$. Measurement of $\mathrm{M_{top}}$ is performed by comparing the m_t^{reco} distribution obtained from the data to these templates using an unbinned likelihood fit.

II. EVENT SELECTION

The dilepton events are selected by requiring two lepton candidates, two energetic jets and large $\not\!\!E_T$.

The data is collected through a central electron and central muon triggers. The electron trigger requires the transverse energy deposition to be greater than 18 GeV. Similarly the muon trigger requires a track with $p_T > 18$ GeV.

Offline Electron candidates are identified as a high-momentum track in the tracking system matched to an electromagnetic cluster reconstructed in the calorimeters with $E_T > 20$GeV. The ratio of hadronic to electromagnetic energy deposition in the cluster is required to be low to ensure validity of the electron hypothesis. Muon candidates are reconstructed as high-momentum tracks with $p_T >$20 GeV/c matching hits in the muon chambers. Energy deposited in the calorimeter is required to be consistent with a minimum ionizing particle. The trigger lepton must be isolated

-we require that energy shared by the towers surrounding the cluster is low. We require that the two leptons have opposite charge.

Jets are reconstructed with the cone algorithm with a radius $R = \sqrt{\eta^2 + \phi^2} = 0.4$. At least two jets with $E_T >$ 15 GeV are required. We only use jets with pseudorapidity $|\eta| < 2.5$.

The missing transverse energy is measured by the imbalance in the calorimeter transverse energy and is required to be greater than 25 GeV.

Additionaly we require that the sum $\mathrm{H_T}$ of transverse energies of all objects in the event and $\not{E}_T$ is greater than 200 GeV.

We reject all events likely to have come from cosmic ray muons, photon conversions or Z boson decays.

Finally we require that the event is succesfully reconstructed by NWA algorithm (*cf.* Sec. III) and that the reconstructed top quark mass m_t^{reco} falls within the range $100 - 320$ GeV. This is done to ensure that the probability density functions are properly normalized.

III. TOP QUARK MASS RECONSTRUCTION

In the $t\bar{t}$ $\rightarrow$dilepton system we do not have enough information to reconstruct the masses of the decaying top quarks. In the detector we measure 4-momenta of jets and leptons and an overall imbalance in transverse energy $\not{E}_T$. However in the dilepton channel we have 2 escaping neutrinos. This means that even when we use all the knowledge of the event such as masses of the particles present and we assume that masses of the two top quarks are the same, we still lack 1 constraint to reconstruct the 4-vectors of final state partons. We will need to integrate over the unknown quantities taking the probability density functions from the Monte-Carlo simulations. In this method we will integrate over pseudorapidities η_1 and η_2 of the two neutrinos. As inputs we will use jets corrected to reflect parton energies, lepton momenta, and $\not{E}_T$. The details have been presented before in [3] and [4]. The essence of the approach is as follows:

- Assume the value of the top mass.
- Choose a particular jet to b-quark assignment (there are two possibilities)
- Assume neutrino pseudorapidities.
- Using the world average masses of the W boson, b quark and leptons, we now can solve for the P_x and P_y of each of the neutrinos. Solutions might not exist for the assumed value of the top quark mass and neutrino η values. When a solution exists we will have two solutions for each neutrino.
- We form four weights comparing each combination of solutions to the measured missing transverse energy with a Gaussian weight. Since the correct combination is not known we sum the four weights.
- We integrate this sum over η_1 and η_2 obtaining the weight for the assumed top mass. The integration distribution for neutrino pseudorapidities is taken from the ttbar Monte Carlo and is a Gaussian with width approximately 1. The integration is performed by summing a grid of η values with 0.2 spacing.
- We obtain the weight corresponding to the other jet to b-quark assignment
- We sum the two weights. Now we have a handle on probability that the true top mass is the top quark mass we assumed.
- We scan the top mass in units of 3GeV.
- The maximum weight is found, as well as maximum weights of the two jet to b-quark assignments separately.
- The scan is repeated succesively around the maxima of the total weight as well as the maxima of the two jet to b-quark assignments with decreasing step size. The search is stopped when step size of 0.03GeV is reached.
- The assumed top mass which yields the highest total weight is taken as the reconstructed top mass m_t^{reco}.

IV. SIGNAL TEMPLATES AND KDE SMOOTHING

We use Pythia [7] $t\bar{t}$ Monte Carlo generated at top quark masses between 120 and 200 GeV/c^2. For each of these samples we obtain the reconstructed mass distribution. To calculate the probability density function for any true top mass $\mathrm{M_{top}}$ as a function of m_t^{reco} we use a Kernel Density Estimate (KDE) method. Advantage of this method is that we do not need to assume any functional form for the probability density function. The estimate of probability density function $\hat{f}(x)$ at any point x is given by the sum:

$$\hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h_i}K\left(\frac{x_i - x}{h_i}\right) \tag{IV.1}$$

where n is number of events in the given Monte-Carlo sample, K is the kernel function, x_i is value of the observable (m_t^{reco}) of the $\mathrm{i^{th}}$ event in the Monte Carlo sample and h_i is the smoothing parameter. The smoothing parameter h_i will be smaller for events in the peak and higher for events in the tail. We use an iterative approach to determine amount of smoothing that will preserve details of the peak of the distribution, but also smooth out the tails sufficiently to remove effects of statistical fluctuations there. First we perform smoothing on a sample with a constant value of h which depends on the sample statistics and width of the peak. This pilot estimate is than used to calculate h_i for all events in the sample.

Form of the kernel function used is:

$$K(t) = \frac{3}{4}(1 - t^2) \text{ for } |t| < 1 \text{ and } 0 \text{ otherwise} \tag{IV.2}$$

Figure 1 shows distribution or m_t^{reco} together with the KDE estimate for several signal samples.

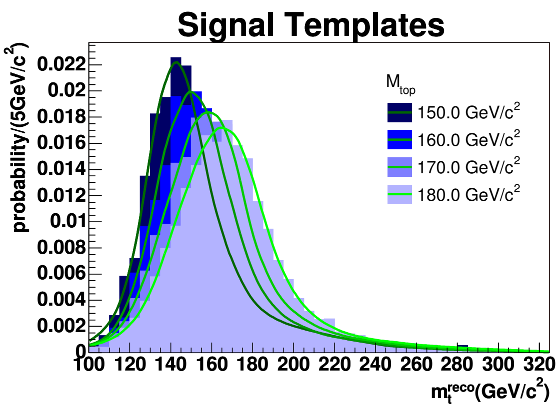

FIG. 1: Distriutions or m_t^{reco} with overlayed KDE estimates for several mass points.

V. BACKGROUND TEMPLATES

The main sources of background in the dilepton channel are fake events where a jet is misidentified as a lepton, the Drell-Yan process, and Diboson production.

To model the fakes background we use a subset of the same dataset as in our measurement. We select events with one lepton and an object likely to be a jet faking a lepton. All other selection requirements are kept. Fakes events are weighted by probability of such an event being a fake. This probability is calculated using QCD enriched samples collected using a jet trigger.

Our Drell-Yan model comes from a matched set of AlpGen+Pythia [6] samples. Included are contributions from $Z \to ee, \mu\mu, \tau\tau, +0, +1, +2, +3, + \geq 4$ partons as well as $Z \to ee, \mu\mu, \tau\tau, +b\bar{b}, +c\bar{c}, 0, +1, + \geq 2$ partons. Both off Z-peak and on Z-peak samples are used. We remove b and c quarks appearing in Pythia showering from light flavor and $Z \to ee, \mu\mu, \tau\tau, +c\bar{c}$ samples. The samples are combined using their relative cross-sections and acceptances.

We model the diboson production precesses using the Pythia generator.

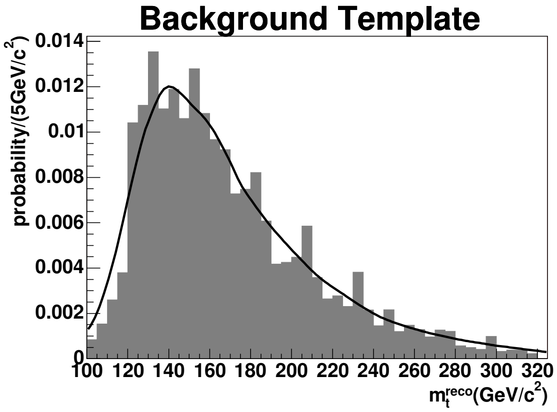

FIG. 2: Combined background probability density function

The KDE estimation proceeds for the background model in the same fashion as for signal. The probability density function is derrived for each subsample taking into account particular sample size and width. The background PDF's are than added with apropriate weights to form the total background pdf shown in Fig. 2.

Table I summarizes expected numbers of events in our dataset.

WW	6.19±1.01
WZ	1.42±0.23
ZZ	0.95±0.75
$Z/\gamma \to$ ee, $\mu\mu$	8.98±2.12
$Z/\gamma \to \tau\tau$	4.33±1.29
Fakes	13.49±5.10
Total Background	35.33±7.00
$t\bar{t}$ ($\sigma_{t\bar{t}}$ = 6.7 pb)	86.80±6.61

TABLE I: Backgrounds and signal estimates for integrated luminosity 1.8 fb^{-1}

VI. LIKELIHOOD FIT

The likelihood form is shown (Eqn. VI.1)

$$\mathcal{L}^{NWA}_{\text{shape}} = \frac{\exp(-(n_s+n_b))(n_s+n_b)^N}{N!} \times e^{\frac{(n_{b0}-n_b)^2}{2\sigma^2_{n_{b0}}}} \times \prod_{i=1}^{N} \frac{n_s P_s(m_t^{reco};\mathrm{M_{top}}) + n_b P_b(m_t^{reco})}{n_s+n_b}, \tag{VI.1}$$

where n_s and n_b are signal and background expectations and N is the number of events in the sample, P_s is the signal probability density function and P_b is the background probability density function. The first term in the likelihood captures the possibility of Poisson fluctuations in the number of observed events. The second term in the product expresses the Gaussian constraints on the background expectation. We use the *a-priori* estimate n_{b0} and its uncertainty $\sigma_{n_{b0}}$ to improve sensitivity. Shape information is used in the third term where probability density functions are used to discern between signal and background events and to extract mass information. We minimize the likelihood with respect to three parameters: $\mathrm{M_{top}}$, n_s and n_b.

From the KDE method we obtain the values of the probability density functions only at the values of $\mathrm{M_{top}}$ where signal Monte Carlo is available. To obtain value of the pdf for arbitrary $\mathrm{M_{top}}$ we use local polynomial smoothing on a per-event basis. A value the pdf is obtained for an event m_t^{reco} for the $\mathrm{M_{top}}$ corresponding to available MC samples. Next a quadratic fit is performed in $\mathrm{M_{top}}$ space where the $\mathrm{M_{top}}$ values far from the required value are deweighted. We take the value of this fit as the value of the pdf $P_s(m_t^{reco};\mathrm{M_{top}})$

VII. METHOD CHECK

We test the procedure by performing 2500 pseudoexperiments at the true top mass values ranging from 150 GeV/c^2 to 185 GeV/c^2. When pseudodata is drawn we fluctuate the background expectation $\mathrm{n_{b0}}$ according to a Gaussian with width $\sigma_{\mathrm{n_{b0}}}$. We further fluctuate the number of background events to be drawn by a Poisson distribution whose mean was obtained in the previous step. Number of signal events to be drawn is selected from a Poisson distribution whose mean is the *a-priori* signal estimate.

The residuals and pull widths from the ensamble tests are depicted in figure 3. We conclude that the method has no bias, however reported error must be scaled up by 2%. This effect is understood to result from limited sample statistics.

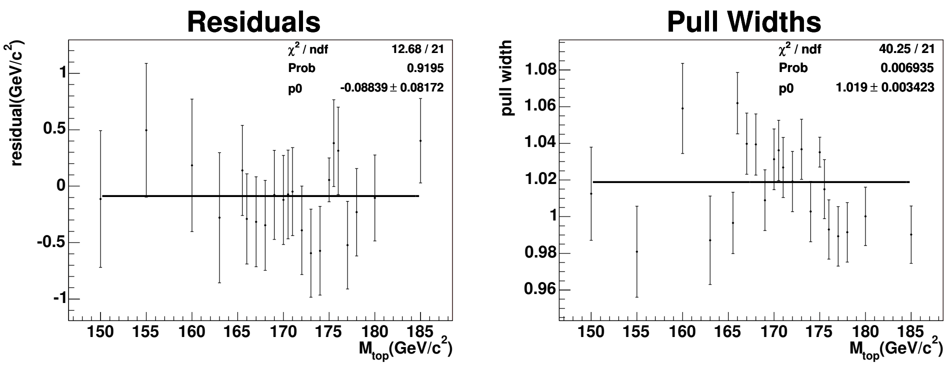

FIG. 3: Residuals and pull widths from ensamble tests as a function of true top mass.

VIII. RESULTS

The likelihood fit when applied to data yields:

$$172.0\ ^{+5.0}_{-4.9}\ \mathrm{GeV}/c^2$$

The fitted signal expectation is $\mathrm{n_s} = 87.3^{+12.8}_{-12.3}$ events. The fitted background expectation is: $\mathrm{n_b} = 36.1 \pm 6.6$ events. The distribution of the reconstructed mass is shown in Fig. 4. Properly scaled background and signal pdf's have been overlaid over the m_t^{reco} distribution. Figure 5 shows a log-likelihood profile in a 15 GeV/c^2 window around the fitted minimum. In this plot the value of negative log likelihood is minimized with respect to $\mathrm{n_s}$ and $\mathrm{n_b}$ at each value of $\mathrm{M_{top}}$. The observed unscaled parabolic error is 4.8 GeV/c^2, consistent with the MINOS errors. In Fig. 6 we show the distribution of parabolic uncertainties in a set of pseudoexperiments performed at $\mathrm{M_{top}} = 172$ GeV/c^2. A smaller error than that observed in the data is obtained 68% of the time.

As a cross check we performed a fit with the background constraint removed. We obtain the same central value with slightly increased uncertainty. We also split the sample by dilepton category. The results are shown in table II. In these fits a background constraint was used constructed using an *a-priori* estimate in these categories.

subsample	Fitted mass	Fitted n_s	Fitted n_b
ee	172.8±12.2	16.68±5.38	10.01±1.92
$\mu\mu$	175.0±8.5	26.78±6.56	13.00±2.60
$e\mu$	169.7±6.5	44.44±8.06	12.66±2.91

TABLE II: Result of fits on subsamples separated by the dilepton category.

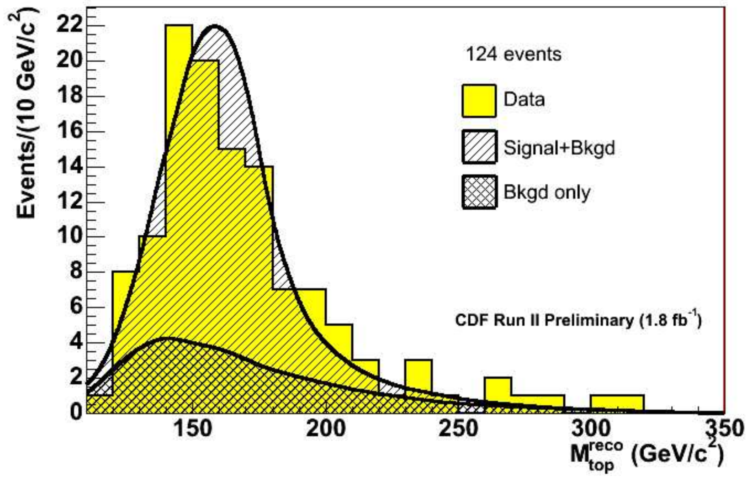

FIG. 4: Distribution of NWA reconstructed mass in data

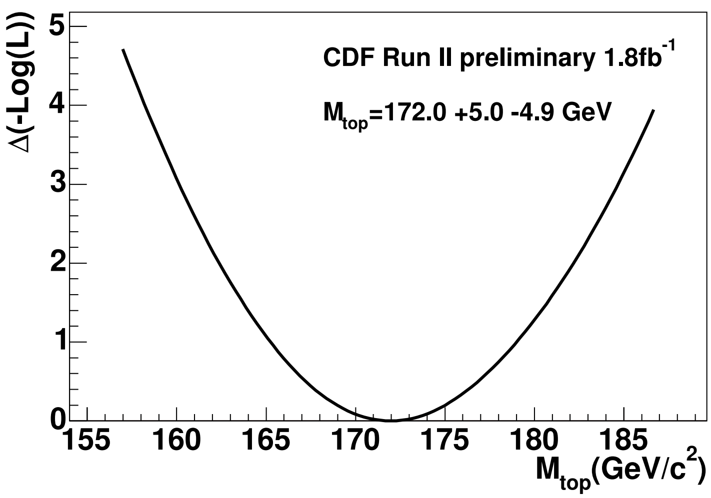

FIG. 5: Likelihood profile in 15 GeV window around the fitted M_{top}

IX. SYSTEMATIC UNCERTAINTIES

The major source of systematic uncertainty is jet energy scale (JES). We attempt to disentangle the major independent effects affecting our modeling. Some of these effects are:

- Relative response of the calorimeters as a function of pseudorapidity with respect to the central calorimeter.
- Single particle response in the calorimeters.
- Fragmentation of jets.
- Modeling of the underlying event energy.
- Amount of energy deposited out-of-cone.

Varying the jets in signal and background pseudodata by the estimated uncertainty on each of those effects and adding the resulting shifts in quadrature yields 3.3 GeV/c^2 systematic effect on the mass measurement.

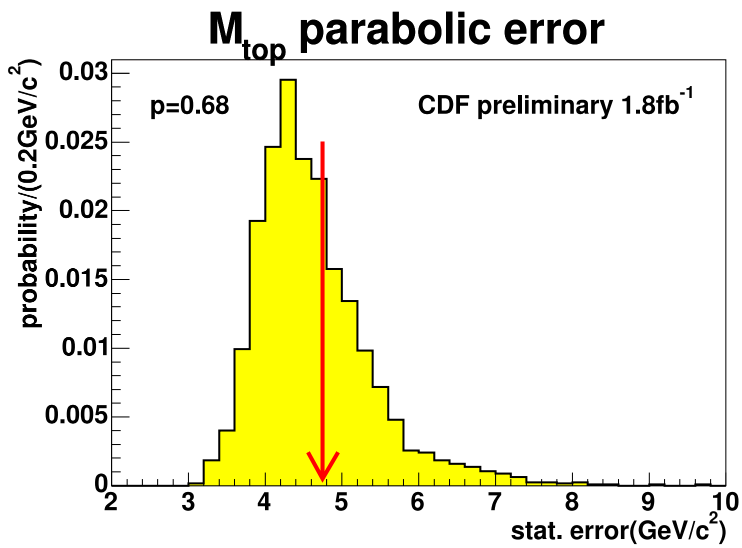

FIG. 6: Expected parabolic error distribution from a set of pseudoexperiments at $\mathrm{M_{top}} = 172\ \mathrm{GeV}/c^2$

The initial and final state gluon radiation is estimated by studying the transverse momentum of Drell-Yan events and extrapolating the results to the Q^2 of a $t\bar{t}$ event. We estimate 0.5 GeV/c^2 and 0.3 GeV/c^2 for the systematic uncertainties due to initial and final state radiation.

Comparison of a sample generated using Herwig [5] generator to that generated by Pythia results in 0.7 GeV/c^2 shift taken as the generator systematic.

We extensively study the effects of the background shape on the final result. We modify the Drell-Yan, Diboson and Fake contributions according to the uncertainties on the *a-priori* estimates for these sanples. We also estimate the size of the fitted mass shift resulting from shift in the Fake event probabilities when the shift is applied in a way expected to maximally correlate with the reconstructed top mass. Finally we study the effect of uncertainty of the composition of the Drell-Yan sample between low and high jet multiplicity samples. The total background shape systematic uncertainty is 0.7 GeV/c^2.

Limited Monte-Carlo sample statistics can result in significant shifts in mean fitted top mass in ensemble testing. This can yield a bias when calibration of the method is studied. This effect is ascertained using the bootstrap method. We estimate 0.1 GeV/c^2 systematic due to signal MC statistics and 0.4 GeV/c^2 due to background MC statistics.

The b jets can behave differently than gluon and light quark jets because of their different fragmentation models, more abundant semi-leptonic decays and different color flow in $t\bar{t}$ events. We find that the uncertainties due to the unique features of the b jet are 0.6 GeV/c^2.

We propagate the estimated 1% uncertainty on lepton energy scale. The effect on the top mass measurement is 0.3 GeV/c^2.

The uncertainties in the parton distribution functions (PDF) are estimated by using different PDF sets (CTEQ5L vs MRST72), different values of Λ_{QCD} and varying the eigenvectors of the CTEQ6M set, yielding a total uncertainty of 0.5 GeV/c^2.

The total systematic uncertainty is 3.6 GeV/c^2. Summary of all systematic effects can be found in table III.

X. CONCLUSIONS

The result of template method top quark mass measurement in the dilepton channel using data sample with integrated luminosity of 1.8 fb^{-1} is:

$$\mathrm{M_{top}} = 172.0\ ^{+5.0}_{-4.9}\ (\mathrm{stat.}) \pm 3.6\ (\mathrm{syst.})\ \mathrm{GeV}/c^2$$

When the uncertainties are combined assuming a Gaussian behavior we obtain:

Source	Systematic (GeV)
JES	3.3
ISR	0.5
FSR	0.3
Generator	0.7
Signal MC statistics	0.1
Background MC statistics	0.4
Background shape	0.7
b-JES	0.6
l-ES	0.3
PDF	0.5
total	**3.6**

TABLE III: Summary of systematic uncertainties

$$\mathrm{M_{top}} = 172.0\ ^{+6.1}_{-6.0}\ \mathrm{GeV}/c^2$$

XI. ACKNOWLEDGEMENTS

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